GmshDDM on LUMI: first runs of a new solver for large scale time-harmonic flow acoustics problems

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LUMI User Day, November 6 2023

LIÈGE université

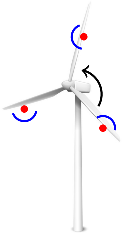
UNIVERSITÉ DE LORRAINE

SIEMENS Ingenuity for life

Industrial context

Predict noise from bodies in motion for the transport/energy industry

Computational (aero)acoustics



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2. Understand sound propagation
3. Find solutions (new material or design)

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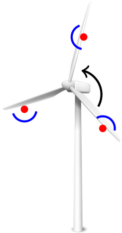

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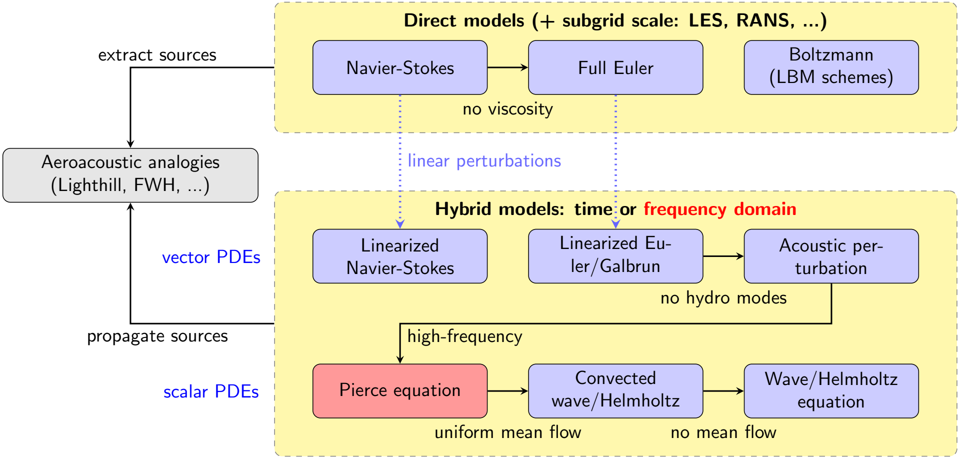
Overall objective

Provide a sound propagation simulation tool

- suitable to modern computer architectures
- applicable to large, complex industrial problems

→ Can serve as basis for optimization routines

Physical models for sound propagation



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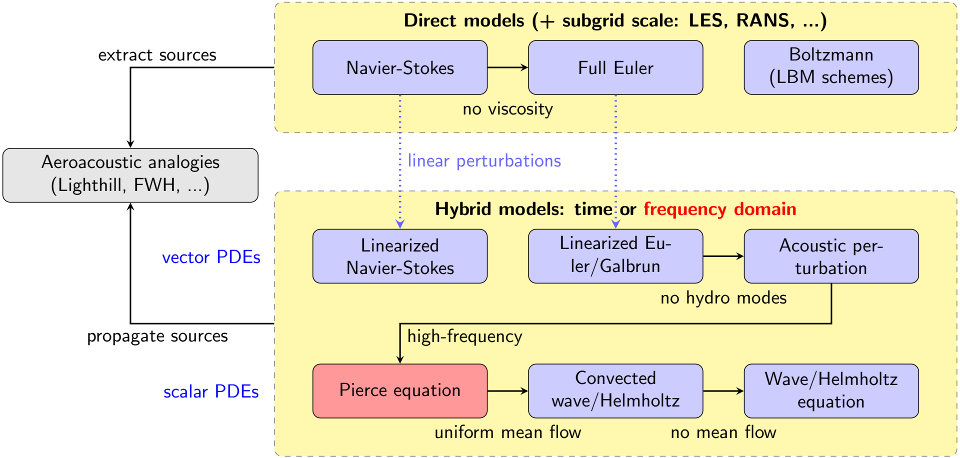


Hybrid model - solve mean flow and acoustic perturbations separately

- We choose the **time-harmonic Pierce Equation**
- Simple but accurate for single tones of turbofan engine intakes and exhausts

Physical model - Pierce Equation

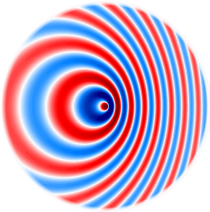
Scalar equation for the acoustic velocity potential u (velocity $\boldsymbol{v} = \nabla u$)

Pierce Equation

$$\rho_0(\boldsymbol{x})\frac{\mathrm{D}_0}{\mathrm{D}t}\left(\frac{1}{\rho_0(\boldsymbol{x})^2 c_0(\boldsymbol{x})^2}\frac{\mathrm{D}_0 u}{\mathrm{D}t}\right) - \nabla\cdot\left(\frac{1}{\rho_0(\boldsymbol{x})}\nabla u\right) = f, \qquad \frac{\mathrm{D}_0}{\mathrm{D}t} := \partial_t + \boldsymbol{v}_0(\boldsymbol{x})\cdot\nabla$$

In frequency domain ($\partial_t \mapsto \mathrm{i}\omega$): Helmholtz-type problem with convection and heterogeneities

Point source in a uniform flow



Mach number $\mathrm{M} = \|\boldsymbol{v}_0\|/c_0 = 0.6$
$\mathrm{M} < 1$ (Subsonic flow)

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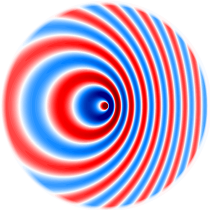

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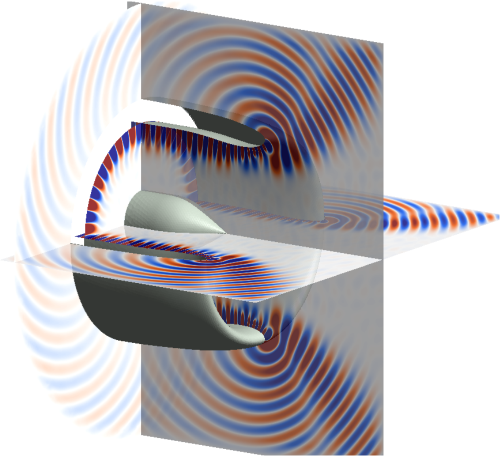
Mathematical difficulties

- Highly oscillatory solution for large ω
- Complex valued, strongly indefinite with ω
- Unbounded domain
- Convection effects

Does not converge with classical iterative methods

Reaching the high frequency limit

State-of-the-art: high-order finite elements (p-FEM) with direct sparse solver (MUMPS)



Turbofan intake at $\omega = \omega_{\text{bpf}}$
(≈ 25 wavelengths)

At $\omega = \omega_{\text{bpf}}$: $N_{\text{dofs}} = 10$M, nnz $= 730$M

Direct solver $\rightarrow$ 740 Gb of RAM

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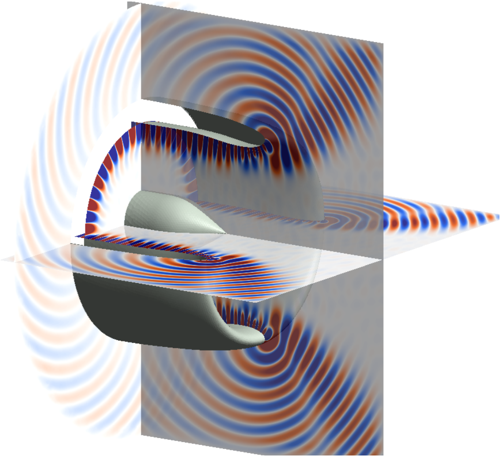

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$\Downarrow$ increase ω?

At $w = 2 \times \omega_{\text{bpf}}$: $N_{\text{dofs}} = 73$M, nnz $=$ 5B

Direct solver $\rightarrow \approx 6$ Tb of RAM

$\mathcal{O}(\omega^3)$ scaling in memory & computational time...

Increasing the upper frequency limit

Scalable domain decomposition method to distribute the memory cost, combining direct and iterative linear solvers

Key ingredients

- High-order finite elements
  - $\rightarrow$ Reduce discretization error (interpolation & dispersion)
  - $\rightarrow$ Increase arithmetic intensity

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- Non-overlapping, substructured optimized Schwarz domain decomposition solver
  - → Sparse direct solver on smaller problems posed on each subdomain
  - → Krylov subspace solver on small number of interface unknowns between subdomains
  - → High-order transmission conditions for quasi-optimal convergence of Krylov solver

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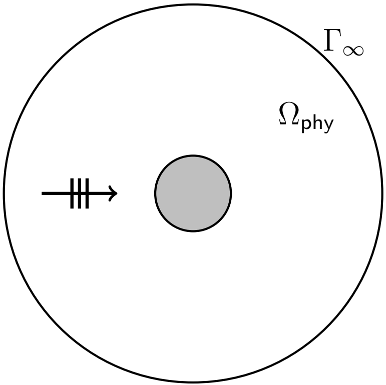
  Good fit for distributed memory cluster architectures

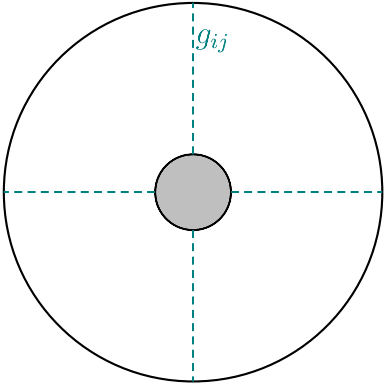
Domain decomposition framework: Non-overlapping optimized Schwarz

Toy example: disk scattering by a plane wave



Domain decomposition framework: Non-overlapping optimized Schwarz

Partition $\Omega = \bigcup_{i=0}^{N_{\text{dom}}-1} \Omega_i$ into subdomains



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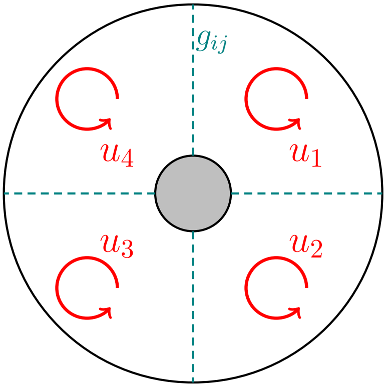

Iterate until convergence

1. Solve the volume subproblems u_i with boundary conditions

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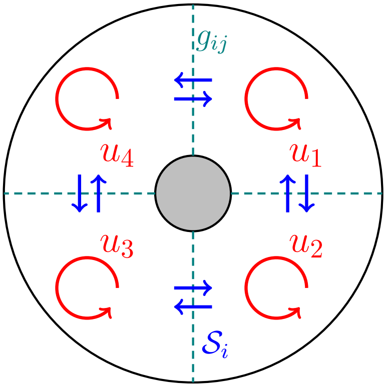

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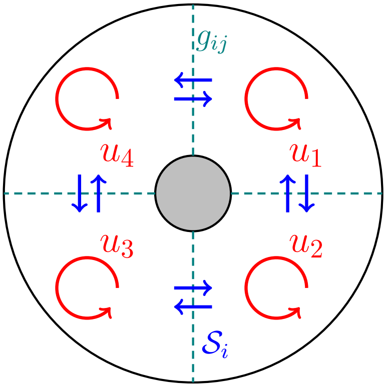

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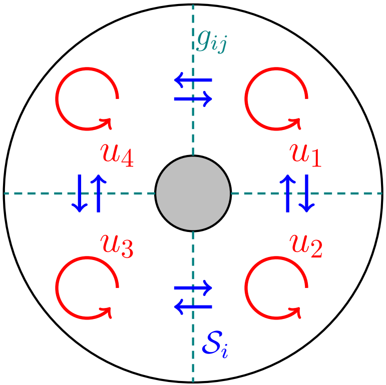

Optimal convergence with the **Dirichlet-to-Neumann** (DtN) operator for the complementary of the subdomain under consideration

We propose a new family of operators $(\mathcal{S}_i, \mathcal{S}_j)$ that provide **accurate approximations of the DtNs in the high-frequency regime**, based on microlocal analysis

GmshDDM: scalable parallel solver for time-harmonic waves problems

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Main features:

- High-order unstructured finite element meshes (1D, 2D, 3D)
- Mesh partitioning using METIS
- Arbitrary order hierarchical basis functions (H^1, H(curl), ...)
- Symbolic specification of weak forms
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Open source under GNU AGPL v3

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Preparatory project, followed by project in **LUMI-BE Regular Access** call

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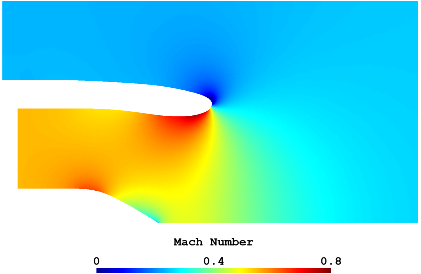
3D turbofan intake and exhaust radiation problems

Given a flight configuration (precomputed mean flow, interpolated on the acoustic mesh), predict the radiated noise

Intake typical mean flow



Compute noise from the fan (fixed annular duct mode), at multiples of the blade passing frequency $\omega_{\text{bpf}}/(2\pi) = 1300$ Hz

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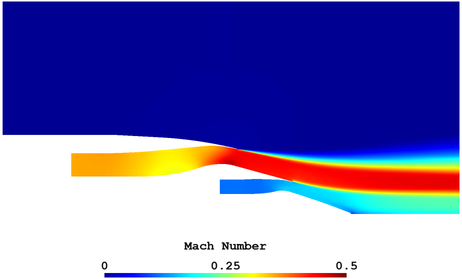

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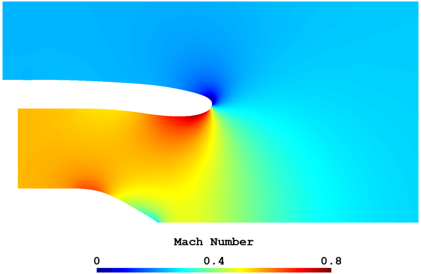

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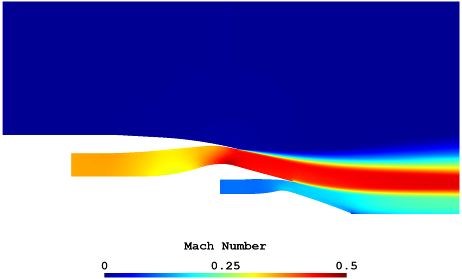

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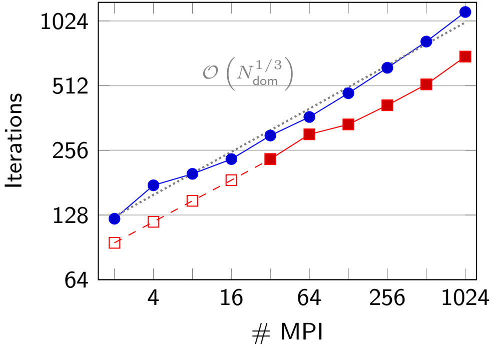
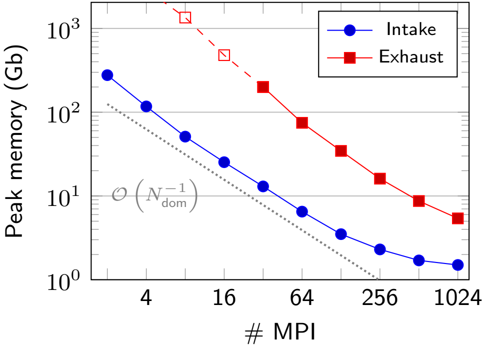
Cylindrical active and passive perfectly matched layers (PMLs) are used to truncate the computational domains

Scalability on small/medium-sized problems

	frequency [Hz]	N_{dofs}	nnz	direct solver est. memory	#tetrahedra
Intake	1300	10M	730M	740Gb	890k
Exhaust	2×7497	86M	6.3B	≈ 10Tb	7.7M



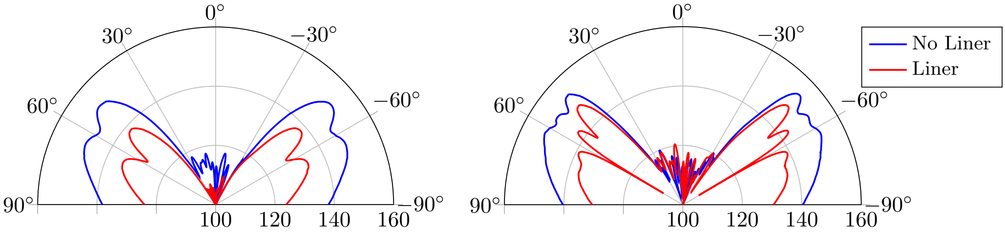
(a) Number of GMRES iterations to reach a relative residual of $r_I = 10^{-6}$.



(b) Peak memory usage over all processes

Engineering results

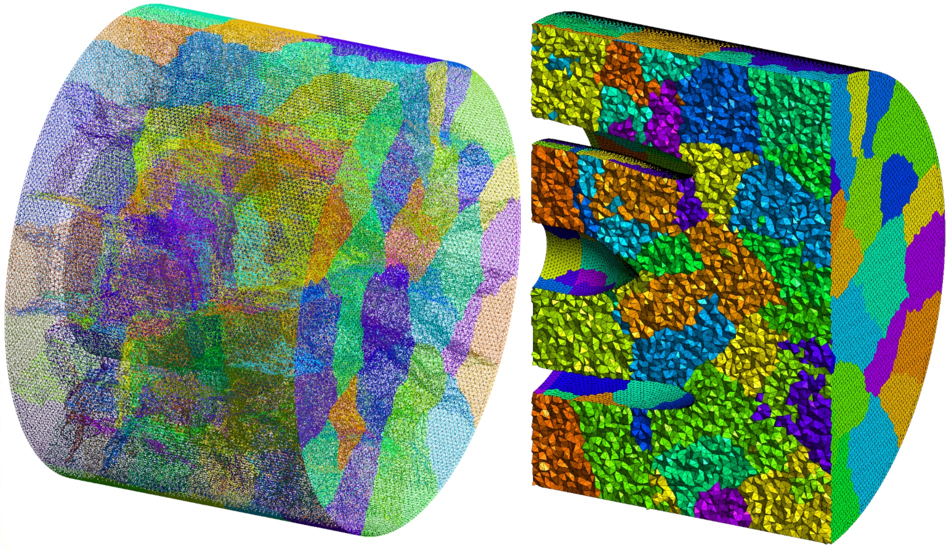
Study of the noise reduction thanks to the acoustic lining



1 BPF   2 BPF

Sound pressure level (dB) near field directivity with and without acoustic lining along a semi-circle of radius 2m centered on the spinner tip in the XY plane.

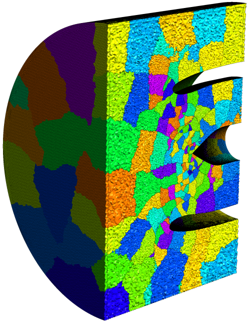
Typical mesh and subdomain partitioning: Intake problem

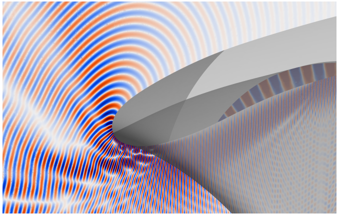


$$N_{\text{dom}} = 512$$

Reaching high frequency: Intake problem at $f = 6500$ Hz (5 BPF!)

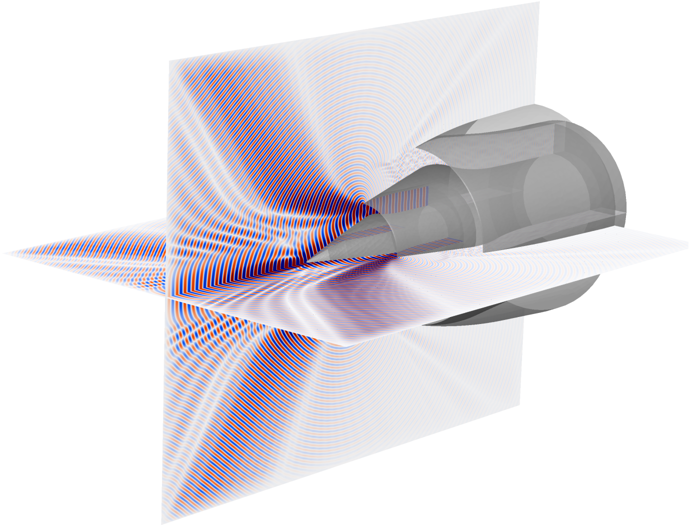
#cores (MPI×threads)	N_{dofs}	nnz	peak mem./process	pre-pro	GMRES	#iter.
1024×64	1.1B	167B	70Gb	24min	3h24min	1293

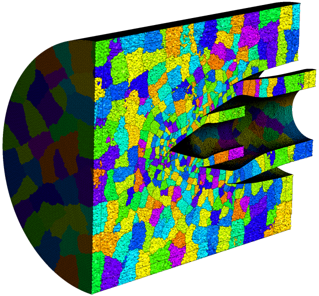


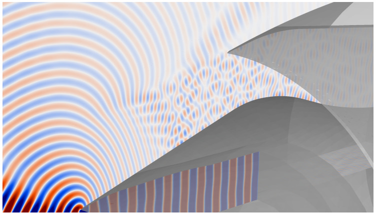


Reaching high frequency: Exhaust problem at $f = 40000$ Hz

#cores (MPI×threads)	N_{dofs}	nnz	peak mem./process	pre-pro	GMRES	#iter.
4096×16	1.3B	96B	18.4Gb	1min	14min	555







Conclusion and perspectives

New distributed memory solver GmshDDM for high frequency flow acoustics

- New quasi-optimal domain decomposition approach
- Performance and scalability validated vs. theoretical bounds
- First LUMI runs allowed to solve large scale problems at unprecedented high frequencies

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Future developments

- GPU acceleration of the iteration phase
  - → Preliminary tests on LUCIA (expected challenge: moving from NVIDIA to AMD GPUs)
- Ongoing work on new applications: electromagnetic waves and elastic waves

References

Open source implementation

- **Gmsh** (`https://gmsh.info`): `https://gitlab.onelab.info/gmsh/gmsh`
- **GmshFEM**: `https://gitlab.onelab.info/gmsh/fem`
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Thanks for your attention

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